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**A parameterized ray-tracing model for evaluation of hemispherical lensing of a uniform, telecentric object aperture – performance improvement estimates for segmented scintillators with miniature lens arrays**

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A parametric model describing the collimation of radiation from a uniform circular aperture by a hemispherical lens of the same radius has been developed for rapid estimation of the potential benefit of miniature dielectric lens arrays in lens-coupled gamma ray cameras (GRCs) with segmented scintillators. A simplified ray-tracing approach is adopted that makes use of small angle approximations relevant to GRCs used in nuclear science at Los Alamos National Laboratory and elsewhere. Application of this analysis technique in a first-pass design optimization process results in the optimistic estimate that miniature lenses fabricated from typical optical quality glass might boost optical coupling by as much as a factor of 4 (6 dB gain). However, this is not likely to be economically attractive compared to other techniques, such as reduction of the GRC lens f-number.

## INTRODUCTION

Efficient coupling of visible light generated within the scintillator to the subsequent detection stages of the gamma ray camera (GRC) is essential to the optimization of quantum efficiency. For a lens-coupled system [Swindell 1991, Liu 1994, Yu 1997], the simplest arrangement possible would have a scintillator material to air boundary (a large step in refractive index) at the optical output side of the scintillator crystal. Some light incident at this boundary is reflected back into the scintillator (and possibly trapped [Shurcliff 1949, Keil 1970]). Light that is transmitted through the boundary is refracted into a broad intensity pattern that is not efficiently captured by the numerical aperture of the lens system. Dielectric matching layers and miniature dielectric lens arrays have both been proposed for lens-coupled GRCs in the hopes that they may provide improvement in optical coupling efficiency. In order to rapidly evaluate the potential benefit of such designs, a simplified ray-tracing model has been developed. This report presents the parametric system of equations and illustrates their use in optimizing and evaluating both strategies—dielectric matching layers and miniature lenses.

## HEMISPHERICAL SUBSTRATE LENSES WITH DISTRIBUTED SOURCES

The utility of an elliptical lens for collimation of a point-source is well-established, and the synthesis of an approximately elliptical lens for implementing gain in microwave and millimeter-wave antenna systems where the point-source approximation is not valid is relatively straight-forward [Buttgenbach 1993, Filipovic 1993]. However, the design and optimization of a miniature lens array for use with a segmented scintillator presents a complication in that the radiating aperture of the scintillator element would be roughly the same size as the miniature lens element, with the majority of the radiation emitted from positions not near the axis of the focusing lens. It is impossible to simultaneously collimate the on-axis differential source while aligning chief rays of off-axis differential sources parallel to the optical axis. Therefore, the optimal improvement in gain is not obvious without some quantitative, even if approximate, analysis.

The strategy implemented in order to obtain an approximation for the far-field power within a given solid angle (the numerical aperture of the lens system coupling the scintillator to the detector) makes use of the fact that the scintillator element is small compared to the overall image and can be treated as a point source in the far-field, i.e. the plane of the coupling lens' limiting aperture. This means that only the ray power and final direction of ray propagation must be retained, not the initial source position within the entrance (object) aperture. If one begins by defining a collection of rays that represent the distribution (position and propagating angle) of power incident at the output plane of the scintillator crystal, the coupled power is approximated by summing over those rays that ultimately fall within the solid angle of the coupling lens. This is easily evaluated for a scintillator pixel near the center of the image plane, and the result serves as an over-estimate for elements near the edges.

The refraction of rays through a parameterized model of the extended hemispherical lens is illustrated in Figure 1. The ray enters the lens with an initial angle of $\theta_{out}$ with respect to the optical axis. At the point where it intersects the lens surface, it makes an angle $\theta_i$ with the surface normal. The ray is refracted through the surface by the well-known Snell's law, and the new angle of propagation may be obtained as $\theta_{transmitted} = \frac{\pi}{2} - \theta_r - \arcsin(n \sin \theta_i)$, where $n$ is the index of refraction of the lens and $\theta_r$ is the elevation angle of the lens surface intersection as shown.

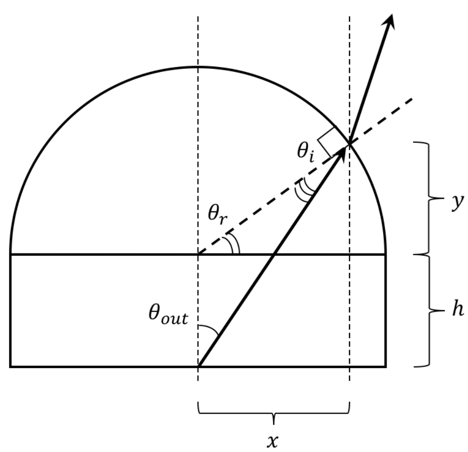


Figure 1. *The refraction of a ray originating from a differential source near the optical axis.*

For completeness, the system of equations that must be solved to obtain $\theta_i$ is:

$$\frac{\pi}{2} - \theta_{out} = \arctan\left(\frac{h+y}{x}\right) \tag{1a}$$

$$\theta_r = \arctan\left(\frac{y}{x}\right) \tag{1b}$$

$$\theta_i + \theta_{out} + \theta_r = \frac{\pi}{2} \tag{1c}$$

$$x^2 + y^2 = r^2 \tag{1d}$$

These four linearly independent expressions are readily obtained from simple geometric principles and easily solved for the 4 unknown variables, $x$, $y$, $\theta_r$, and $\theta_i$. The results of this ray-tracing are given in Figure 2 for an example lens design. An LSO crystal (n=1.92, r=0.505 mm) is coupled to a glass lens (n=1.47) of the same spherical radius. The extension of the hemisphere, $h$, is 0.8 mm so as to mimic a collimating ellipse. Part a) shows the intensity per unit solid angle for the visible radiation field that is assumed to be incident on the crystal/lens boundary (blue line), that which is refracted into the lens (orange line), and that which is transmitted from the lens into air (green line). The intensity field that would be observed if no lens were present is shown in red. Part b) displays the power and propagation angle of each ray bundle used for the calculation. Each data point represents an expanding cone of rays that appear in the far-field to have originated from a point source.

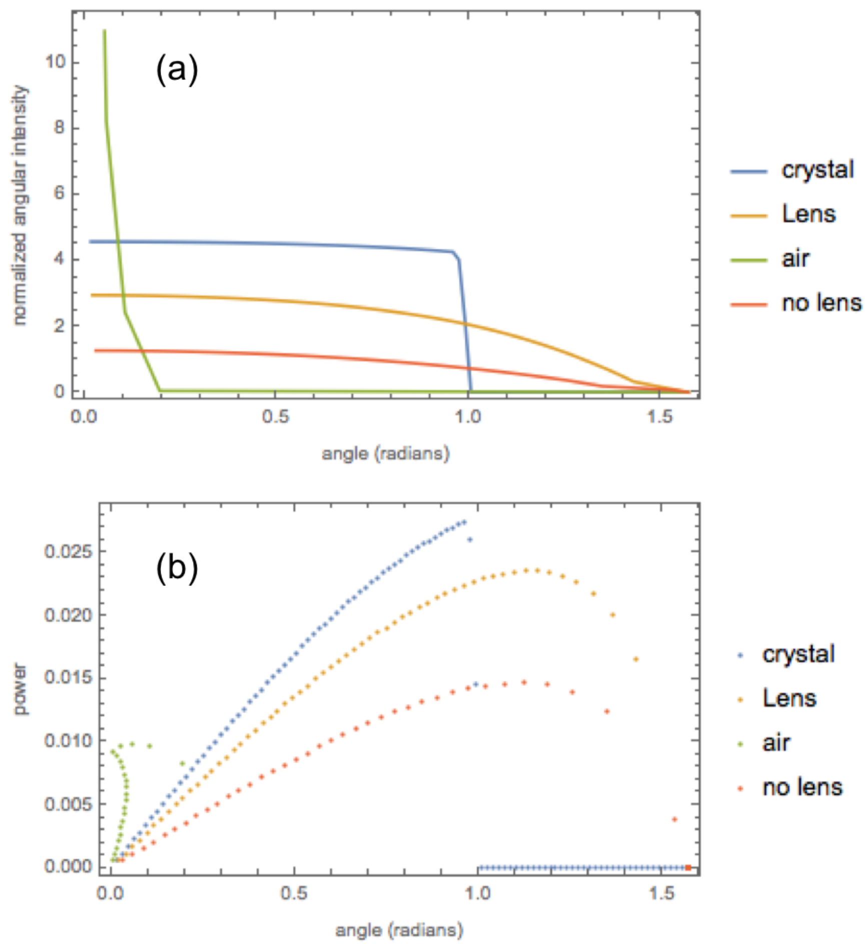


Figure 2. a) *The intensity field distribution assumed at the optical output plane of the scintillator crystal, refracted into lens material, and refracted by a lens surface into air. Also shown is the result without a lens.* b) *The power and propagating angle of conical ray sections used in the calculation. Green dots indicate rays well-collimated along the optical axis by the lens.*

The far-field distribution produced by off-axis sources is approximated by steering only the chief ray and assuming that the sagittal and meridional rays are refracted in the same manner relative to the chief ray as they were in the on-axis source case. This is a crude approximation for coupling lenses with very small f-number, but in the small-acceptance-angle limit of a moderate f-number lens (such as is typical for GRCs used in nuclear science) it has a modest impact on the estimation of coupling efficiency. This is due to the fact that sources with chief rays refracted to large angles, and hence exhibiting large distortions in their far-field radiation pattern, are not considered in the final summation. The accuracy to which the direction of propagation for these rays is evaluated is therefore irrelevant.

The geometry considered for evaluating the refraction of a chief ray emanating from a telecentric differential source away from the optical axis of the miniature lens is illustrated in Figure 3 below. Again, the direction of propagation is obtained as $\theta_{trans} = \theta_r + \arcsin(n \sin\theta_i) - \pi/2$. The system of equations used to obtain $\theta_i$ is:

$$y = \sqrt{r^2 - x^2} \tag{2a}$$

$$\theta_r = \arctan\frac{y}{x} \tag{2b}$$

$$\theta_i = \frac{\pi}{2} - \theta_r \tag{2c}$$

In this system, there are only 3 unknown variables; $x$ is pre-defined.

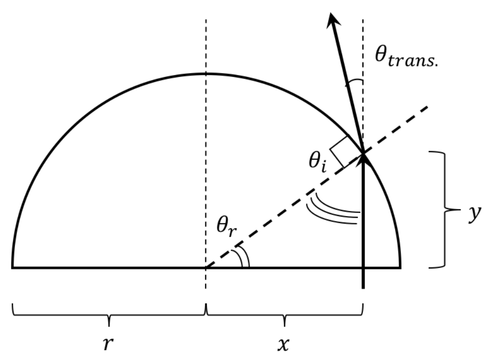


Figure 3. *The refraction of a chief ray emanating from a source not centered on the optical axis.*

Having approximated the direction of transmission in the far-field for all differential elements on the circular source plane, all that remains is to sum the power contained in those rays that fall within the solid angle subtended by the coupling lens' limiting aperture. This is trivial for the on-axis differential source element. However, each off-axis differential source element produces a cone of rays intersecting a circle that is displaced on the far-field unit sphere. A finite fraction of this circle falls within a region defined by the coupling lens' acceptance angle. This is illustrated in Figure 4, where a small-acceptance-angle approximation allows one to represent the surface of the unit sphere as a plane. On this planar approximation, the intersecting circles have radii $\theta_{accept}$ and $\theta_{trans}$. They are displaced by a distance determined by the angle of the off-axis differential element's chief ray, here referred to as $\theta_{chief}$. The fraction of the circle of rays that fall within the region defined by $\theta_{accept}$ is $\Omega_{coupled}/\pi$, where the angle $\Omega_{coupled}$ is obtained using the law of cosines:

$$\Omega_{coupled} = \arccos\left(\frac{\theta_{chief}^2+\theta_{trans}^2-\theta_{accept}^2}{2\theta_{chief}\theta_{trans}}\right). \tag{3}$$

The power contained in an annulus of off-axis differential sources is multiplied by this fraction to obtain a differential coupled power. A summation over all differential annular sources produces the total coupled power.

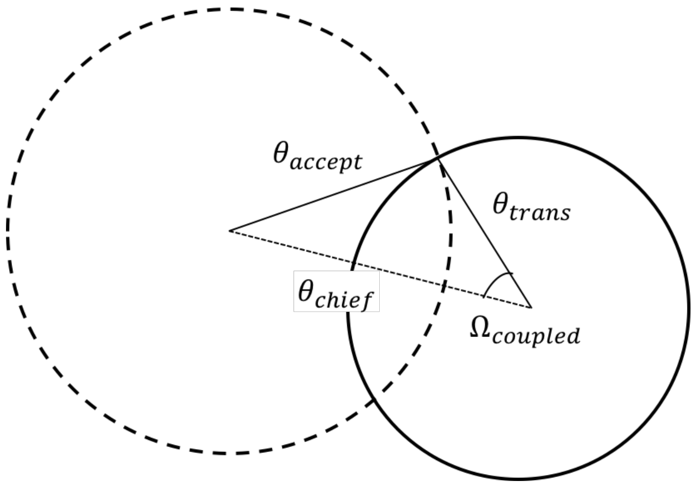


Figure 4. *The intersection of a circular bundle of rays with the coupling lens' acceptance aperture on a planar representation of the far-field unit sphere.*

## PERFORMANCE IMPROVEMENT ESTIMATES FOR DARHT-SIMILAR SYSTEMS

Given its approximations, the analysis described in the previous section provides a very crude estimate of the coupling efficiency for a realistic scintillator array. It is in no way a substitute for more sophisticated analysis that can be accomplished with commercially available software, but can be taken as an idealized, best-case scenario for rapid cost benefit analysis. In all cases shown below, the acceptance half-angle of the coupling lens is taken to be 5 degrees, similar to DARHT Axis I.

The simplest arrangements to evaluate include the addition of planar dielectric matching layers. The result of modeling a two-layer system is shown in Figure 5. Part a) compares the intensity pattern that arrives at the crystal/air boundary with the pattern of intensity refracted into air when no matching layer is present. Part b) shows the intensity field as it propagates through a set of two matching layers. Layer 1 may be a high-refractive index polymer (HRIP), and is taken to have n=1.7. Layer 2 is similar to a typical optical quality glass, having n=1.5. The field that arrives in air is identical to that obtained in Part a). Index matching is a zero-sum game for an array of this type; the power within a given solid angle is not improved so long as the radiation must ultimately propagate into air.

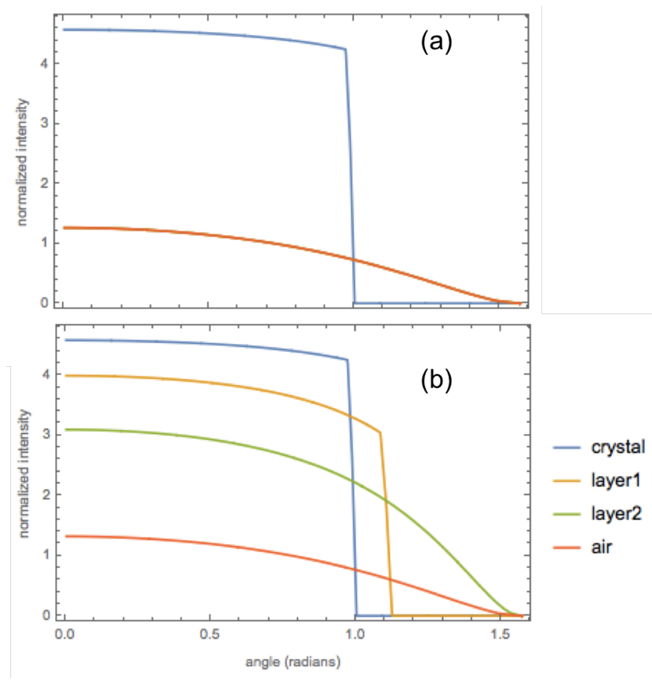


Figure 5. a) *The visible photon intensity pattern assumed just inside a crystal/air boundary (blue) compared with that obtained in air (red).* b) *The propagation of the same field (blue) through a high-index material (n=1.7), a typical glass (n=1.5), and finally into air. The scintillator crystal is taken to be LSO with n=1.82.*

Estimates for the gain provided by collimation through an extended hemispherical lens element are given in Figure 6. Results for 3 different possible indices of lens material refraction are plotted vs. the stack height of the lens, *h*. Note that in the high index case (n=1.8), the stack height has little meaning since the plane of crystal/lens interface is poorly defined—it is an infinitesimal step in refractive index. For the other two cases, there is a physically meaningful optimum stack height that provides improvement in collimation of radiation originating near the crystal axis without excessive deleterious refraction of off-axis radiation. As expected, the optimal stack height is inversely proportional to the refractive index of the lens element.

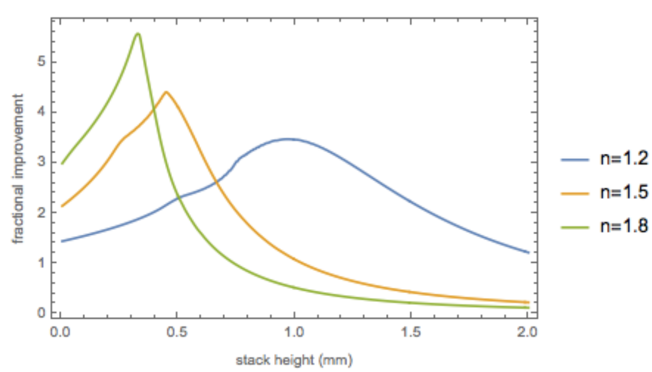


Figure 6. *Estimates for the improvement in gain provided by a hemispherical lens, plotted vs. stack height for 3 different hypothetical lens materials.*

## SUMMARY AND CONCLUSIONS

A simply analysis method for evaluating the potential benefit of adding a miniature lens array to the segmented scintillator of a DARHT-like GRC has been developed with the following approximations: scintillator elements are cylindrical with uniform and nearly Lambertian optical output, spherical aberration and other distortions are unimportant within the small acceptance angle of the coupling optics, and the acceptance angle is sufficiently small on the far-field unit sphere so as to be represented as a planar surface. Subsequently, this model provides an optimistic estimate for the improvement to be had in coupling an idealized scintillator pixel near the center of the array. If a miniature lens array were to be pursued, this analysis would in no way serve as a substitute for more realistic modeling. However, it has been performed here for rapid first-pass cost benefit analysis.

While the addition of dielectric matching layers does not improve the intensity field pattern or coupling to the GRC's detector plane, there is improvement to be had by lensing of the individual scintillator pixels. Estimates here are inherently optimistic, but predict a factor of 4 improvement when a typical optical quality glass is used. However, this roughly 6 dB of gain is not likely to be economically attractive given the complication and cost associated with designing and fabricating such an array.

Obvious complications in designing a realistic array include not only the necessity of more comprehensive computational modeling, but also the mitigation of element-to-element optical coupling within the lens array that would induce additional blur and the treatment of pixels not near the center of the array. The latter issue would require some variation in the miniature lenses from element to element, at very least a careful shift in position relative to the scintillator crystal. Given that a DARHT or GRC application would require miniature lenses much, much larger than those fabricated by industry for other applications (such as LED displays), there is no standardized method available to manufacture such an array. Individual lenses would be costly from both a fabrication and assembly standpoint due to the large number of pixels (~150,000). Cutting or etching a glass array would also become costly, and an array molded of polymer material would be subject to radiation darkening over time. Fortunately, refractive index matching is not strictly necessary for gain optimization. Nonetheless, the cost of a miniature lens array, particularly compared with the improvement in coupling that is possible by increasing the numerical aperture of the coupling lens, makes it unattractive at the present time in light of the modes gain it might provide.

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